SRI PADMAVATI MAHILA VISVAVIDYALAYAM::TIRUPATI

Master of Science (Mathematics) Third Semester

ASSIGNMENT

M.Sc.MD 3.01 – FUNCTIONAL ANALYSIS

All questions carry equal marks

4 x 5 = 20 Marks

1) Let N and N' be norm linear spaces and T be a linear transformation of N into N'. Then show that following conditions on T are equivalent to each other

(a) T is Continuous

(b) T is Continuous at origion ie $x_n \to 0 \Rightarrow T(x_n) \to 0$.

(c) There exists a real no. $K \geq 0$ with the property that $\|T(x)\| \leq K\|x\| \ \forall x \in N$.

(d) If $S = \{x : \|x\| \leq 1\}$ is the closed unit sphere in N then its image $T(S)$ is bounded in N'.

2) State and prove uniform Boundedness theorem.

3) Let H be Hilbert space and $\{e_i\}$ be an orthonormal set in H. Then show that the following conditions are all equivalent each other

(a) $\{e_i\}$ is Complete

(b) If x is an arbitrary vector in H such that $x \perp \{e_i\} \Rightarrow x = 0$.

(c) $x = \Sigma (x_i e_i) e_i$

(d) $||x||^2 = \sum_i |(x_i e_i)|^2$

4) If P is a projection on H with range M and null space N. Then show that $M \perp N$ if and only if P is self adjoint. In this case $N = M^{\perp}$.

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M.Sc.MD 3.02 – ADVANCED COMPLEX ANALYSIS

All questions carry equal marks

4 x 5 = 20 Marks

1) State and prove Laurent's theorem.

2) (a) State and prove Residue theorem

(b) Evaluate $\int_0^{2\pi} \frac{d\theta}{a+b\cos\theta} = \frac{2\pi}{\sqrt{a^2-b^2}}$

3) (a) State and prove Argument principle

(b) State and prove Rouche's theorem.

4) State and prove Weierstrass theorem.

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Master of Science (Mathematics) Third Semester

ASSIGNMENT

M.Sc.MD 3.03 – INTEGRAL TRANSFORMS

All questions carry equal marks

4 x 5 = 20 Marks

1) (a) Find the Fourier transform of $f(x)$ defined by $f(x) = \begin{cases} 1 & |x| \leq a \\ 0 & |x| > a \end{cases}$ and hence evaluate

(i) $\int_{-\infty}^{\infty} \frac{\sin sa \cos sx}{s} ds$ (ii) $\int_{0}^{\infty} \frac{\sin s}{s} ds$.

(b) Show that $\int_{0}^{\infty} \frac{\cos \lambda x}{\lambda^2+1} d\lambda = \frac{\pi}{2} e^{-x}, x \geq 0$

2. State and prove Fourier Integral Theorem.

3. Convert the integral equation $F(t) = 1 + \int_{0}^{t} F(u) \sin(t-u)\, du$ and verify your solution

4) use the method of Fourier tranform to determine the displacement $y(x,0)$ of an infinite string given that the string is initially at rest and the initial displacement is $f(x)$, $-\infty < x < \infty$, show that the solutions can also be put in the form

$$y(x,t) = \frac{1}{2}\left\{f(x+ct) + f(x-ct)\right\}.$$

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Master of Science (Mathematics) Third Semester

ASSIGNMENT

M.Sc.MD 3.04 – MATHEMATICAL METHODS

All questions carry equal marks

4 x 5 = 20 Marks

(1) Expand in a Series of Sines and Cosine multiples of function given by $f(x) = \begin{cases} x-\pi, & -\pi < x < 0 \\ \pi - x, & 0 < x < \pi \end{cases}$ What is the sum of the series at $x = \pm\pi$ and $x = 0$.

(2) Find the Solution of Brachistochrone Problem.

(3) Define Green's function and mention its properties

(4) Solve $y(x) = 1 + \int_0^1 y(t)\,dt$

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Master of Science (Mathematics) Third Semester

ASSIGNMENT

M.Sc.MD 3.05 – FLUID MECHANICS

All questions carry equal marks

4 x 5 = 20 Marks

1. (i) Derive Euler's equation of motion of an inviscid, in compressible fluid.
   (ii) State and prove kelvion's Circulation theorem.
2. State and prove Milne Thomson circle theorem.
3. Derive the relation between stress and rate of strain.
4. Discuss the steady flow through a tube of uniform circular cross-section (Poiseville flow)